Decoding Chinese Calendars in the Qin and Han Dynasties

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1. Background

In the official history of empire China, the first calendar-making system existed today appears in the *History of Han* 《汉书》. This calendar was composed in 104 BC. Before then, the only message we know is that people made use of a *sifen li* (*quarter calendar system* 四分历) to compute the calendar table. Here *quarter* refers to the remainder, 1/4 days, of a tropical year used in this system. Others are not so sure.

Therefore, in the last 2000 years, what the calendar was before 104 BC has been a big problem to historians. Many reconstructed programs appeared, of course, more or less different to each other. Since 1950s, archaeological excavations have made huge progress in China. A lot of bamboo slips in pre-Han dynasty were unearthed. Some records in these first hand historical literatures tell us useful information of pieces of the calendars tables before 104 BC.

According to these materials, historians checked the calendar tables we already have had in hand, or reconstructed the calendar-making system in the Qin 秦 (246BC – 207BC) and Han 汉 (206BC – 105BC) dynasties from time to time. The interesting thing is that, not surprising to us, new reconstructed system is always denied by the next batch of materials unearthed.

In this short note, we want to develop a tool to analysis the mathematical structure of Chinese calendar-making system, and decode its secret in the Qin and Han dynasties. Moreover, this method may be used to check the calendars before the *Linde li* (*a calendar-making system used in the Linde period* in the Tang dynasty, 麟德历, 664AD).

2. Postulates, definitions and notions

It is well known that Chinese calendar-making system is a lunar-solar system. One will see that such a calendar system could be taken as a coding system. Let CCC system stand for Chinese Calendar Coding system.

As we mentioned as above, our aim is to create a new technique which could be used to analysis a Chinese calendar-making system. We hope this technique could help us to decode a CCC system composed in old China, for instance, the CCC system in the Qin and Han dynasties. For doing this, let's start from the following postulates.

Postulate 1. The calendars issued by the Qin and Han dynasties during the period from 246 BC to 105BC were calculated by some calendar-making systems.

Postulate 2. The length of a tropical year is 365.25 days. A calendar year has 12 calendar months. The cycle of leap month being used in these systems is that 7 leap months will be added in 19 tropical years. It yields that the synodic month is 29+499/940 days.

All historical materials, such as the *History of Han* 《汉书》, support the Postulates above. For making things clearer, we need to give the following two definitions before we introduce some necessary concepts.

Definition 1. Assume the series of a Grand Cycle of a calendar is *G* which is a closed chain. Any point of *G* could be the starting point of a calendar. If *G* could be cut into *n* pieces in the same length $|C|$, i.e. $|G| = n\,|C|$, and the series of these pieces equals to each other, *C* is called a sub-cycle of *G*.

Definition 2. *G* is a closed chain in *days*. *G** is a closed chain in *years*. A grand cycle, *G* or *G**, is called a code dictionary. The structure of a code dictionary is determined by the series of calendar-month. When we add the rule of calendar-year to the dictionary, at the same time, choose a point at the closed chain *G* or *G**, the dictionary is called a coding system.

A CCC system is a *Book* of days, months and years. This *Book* consists of a few kind of cycles in different levels, i.e. a big cycle consists of some sub-cycle. Here are some related notions in a CCC system:

Ganzhi 干支 (stems and branches)**:** a cycle of 60 by paring off two series of terms for numbering the days and years in order. There are 10 "heavenly stems" (*tiangan* 天干：甲、乙、丙、丁…) and 12 "terrestrial branches" (*dizhi*, 地支：子、丑、寅、卯….).

Day (Letter): the days and years adopt the same *sexagenary series*, namely *Ganzhi*.

Month (Word): a calendar-month consists of either 29 or 30 days. The average length of calendar-month equals to a synodic month, roughly say 29.5306 days.

Year: a normal calendar-year consists of 12 months. The average length of calendar-year equals to a tropical year, roughly say 365.2422days. One more month, namely leap month, will be added into a calendar-year in every 2 or 3 years so that the calendar-year and tropical year could be matched.

Section @: the calendar-month as 29 or 30 days alternates strictly in the series. While two big months (30 days) appear consecutively in the series, we devote @ to the later one. Each @ ends a section.

Chapter *C*: a chapter *C* consists of a certain sections. *C* is a sub-cycle of the grand cycle.

Volume *G*: a volume *G* consists of a certain sections *C*. In a volume *G*, the initial letter will be getting back to the same *ganzhi*, i.e.

$$|G|_d = 0 \pmod{60}$$

Book *G*:* a book *G** consists of a certain volume *G*. *G** is a code dictionary. In a book *G**, the initial year will be getting back to the same *ganzhi*, i.e.

$$|G^*|_y = 0 \pmod{60}$$

3. Some results

Let the fraction of a synodic month be n / m days. We always suppose that $m < 2n < 2m$, and $(n, m) = 1$. It is easy to obtain the following results.

Proposition 1. Let $k = \left[\frac{m-n}{2n-m} \right]$. Then $|A| = 2k+3, \quad |B| = 2k+1$.

Example. If $n / m = 499 / 940$, there are $|A| = 17, \quad |B| = 15, \quad k = 7$. The series of section *A* and *B* are as follows:

A: = 0 1 0 1 0 1 0 1 0 1 0 1 0 1 0 1 @

B: = 0 1 0 1 0 1 0 1 0 1 0 1 0 1 @

Where 0 and 1 stand for the small month (29 days) and big month (30 days), respectively. Please note that @ also stand for a big month. The role of section *A* and *B* in *G* is similar to the genetic code (*ATGC*) in the DNA double helix.

Proposition 2. Let $r = (m-n) - k(2n-m), \quad r + r' = 2n - m$. Then

$$|C| = r|A| + r'|B| = m.$$

Example. If $n / m = 499 / 940$, there is $|C| = 35|A| + 23|B| = 940 \text{ months}$. The series of chapter *C* is as follows:

A ABAAB ABAAB ABAAB …… ABAAB AB

The chapter *C* could be taken as a piece of genetic chain *G* which features the structure of a calendar. We can use this series to check the reconstructed calendars in the Qin and Han dynasties. If a reconstructed calendar table does not match with the chapter *C* in a certain period, it means that either this table was not calculated by the calendar-making system in the Qin and Han dynasties, or it was produced by more than one such calendar-making systems (the same system with different epochs).

Proposition 3. Let $@_p$ be the remainder of the *p*-th section in a series of *C*. Then

$$@_p \equiv r' p\left(\mathrm{mod}\left(2n - m\right)\right).$$

Example. If n / m = 499 / 940, then there is

$$@_p \equiv 23p\left(\mathrm{mod}58\right).$$

$@_p$ could identify the exact position of the *genetic code A* or *B* at a *genetic chain C*.

A_{23}	A_{46}	B_{11}	A_{34}	A_{57}	B_{22}
	A_{45}	B_{10}	A_{33}	A_{56}	B_{21}
	A_{44}	B_{9}	A_{32}	A_{55}	B_{20}
	A_{43}	B_{8}	A_{31}	A_{54}	B_{19}
	A_{42}	B_{7}	A_{30}	A_{53}	B_{18}
	A_{41}	B_{6}	A_{29}	A_{52}	B_{17}
	A_{40}	B_{5}	A_{28}	A_{51}	B_{16}
	A_{39}	B_{4}	A_{27}	A_{50}	B_{15}
	A_{38}	B_{3}	A_{26}	A_{49}	B_{14}
	A_{37}	B_{2}	A_{25}	A_{48}	B_{13}
	A_{36}	B_{1}	A_{24}	A_{47}	B_{12}
	A_{35}	B_{0}			

At the moment, we omit the role of years in a grand cycle *G**. If one employs the property of years combined with the propositions mentioned above, it could be easier to identify the fragment of terms to the original coding system.

4. Remarks

Proposition 2 gives the code chain of a CCC system. Proposition 3 tells us how to compute such a code chain. Starting from the two Postulates, one can easily compute the code chain of a *sifen* CCC system, see the examples above. All *sifen* CCC systems have the same code chain. The only different from one to others is its epoch. With the help of the information unearthed from time to time, one can calculate all possibilities of epochs which the CCC system we are decoding could be.

Moreover, a CCC system could be taken as a coding system. This system has the following properties:

Every point of the closed chain, G or G^*, will form a coding system. The starting point is called epoch of this coding system.

The dictionary G or G^* is opened to public. But the epoch one chose should be always keeping as a secret among interior. Decoding a CCC system is equivalent to find the epoch.

Since the days and years adopt the same sexagenary series (*ganzhi*), fragments that people obtain are some of the terms of *ganzhi*. The same terms in different places (days) are different to each other. They are different because their remainders are different.

The most important character is that one communicates, by using the coding system, simply with the TERMS (*ganzhi*). The REMAINDERS attached to their terms keep a secret to open.

The fragments of terms (*ganzhi*) may identify the coding system. But it seems that it is impossible if one could not obtain enough information of the TERMS. In this sense, we say that a CCC system may provide a new system of the coding theory.

Decomposing the Grant Cycle of *quarter calendar system*

$\underline{1\text{st} \quad 2\text{nd} \quad 3\text{rd} \quad \cdots \quad 29\text{th}}$ 1st 2nd $\cdots$ 30th 1st $\cdots$

$$|0| = 29 \text{ days} \quad |1| = 30 \text{ days}$$

$\underline{0 \quad 1 \quad 0 \quad 1 \quad \cdots \quad 0 \quad 1 \quad @}$ 0 1 $\cdots$ 0 1 @ 0 1 $\cdots$

$$|A| = 8|0| + 9|1| \quad |B| = 7|0| + 8|1| \quad @ = 1$$

$\underline{\Lambda\ ABAAB \cdots ABAAB\ \ AB}$ $\Lambda\ ABAAB\ ABAAB \cdots$

$$|C| = 35|A| + 23|B| \quad \Lambda = A$$

$\underline{C \quad C \quad C \quad \cdots \quad C}$

$$|G|_d = 20|C| = 20 \times 27759 \text{ days} \equiv 0 \pmod{60}$$

$\underline{G \quad G \quad G}$

$$|G^*|_y = 3|G|_y = 4560 \text{ years} \equiv 0 \pmod{60}$$