

Historical Astronomy and Eclipses

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Abstract

In this report, we first review the history of the study for the determination of the long-term variations of the Earth rotation and the accelerations of the lunar motion, which were once called the solar and lunar accelerations (including the accelerations of the planetary motions). In the second part of the report, we describe the present status of the study focusing on our works.

1 Introduction

1.1 ΔT

The delay time ΔT of the Earth rotation is defined by

$$\Delta T = \text{TT} - \text{UT}$$

where TT (Terrestrial Time) denotes the homogeneously flowing time, and UT (Universal Time) is the time measured by the rotation of the Earth. ΔT has been adjusted to zero in the 19th century, and the **rate** has been also adjusted then.

ΔT increases if we go back to the past. This reflects that the Earth rotation is secularly slowing down due to the tidal interactions with the moon. The Earth rotation was faster in the past. We want to determine long-term variations of ΔT .

1.2 Determination of long-term variations of ΔT

Variations of ΔT reflects, on the one hand, the secular decrease of the Earth's spin angular momentum due to the conservation of the angular momentum of the Earth–Moon system. The Moon is receding from the Earth by 3.8cm/year due to the tidal friction, which results in the increase of the orbital angular momentum of the Earth–Moon system. Then the spin angular momentum of the Earth decreases. Variations of ΔT reflects, on the other hand, the irregular variations of the inertial moment of the Earth. The larger the inertial moment, the smaller the spin rate.

The ancient observational data indicates the existence of the uplift of the sea level backwards in time at least until 2700 years ago (see the discussions in Sect. 3.2). In

addition shorter variations of ΔT are expected. In order to obtain the shorter variations, we need to analyze time-dense ancient data with small errors.

We collect ancient data of various kinds from various countries. There are solar and lunar eclipses. There are timed and untimed records. There are total, annular and partial eclipses. There are records with comments and without comments. There are records of occultations of planets and bright stars by the Moon. There are observations of equinoxes, and meridian passages of Mercury and Venus.

There is a long history of studies of lunar and solar accelerations. Let us briefly review the history of the past studies.

2 Studies of solar and lunar accelerations

2.1 The early interest

Until the early twentieth century, the slowing down of the rotation of the Earth was not known. Astronomers thought that the Earth was rotating with constant speed. Here, let us explain what happens to the observations of the motion of heavenly bodies (the Sun, the Moon and planets) if one assumes that the rotation of the Earth is constant contrary to the fact that the rotation of the Earth is slowing down secularly. If we measure the time by the rotation of the Earth, then,

1. The unit of time grows. As a result,
2. The motion of the Sun accelerates.
3. The motion of the Moon accelerates.
4. The motion of planets accelerates.

In this case, the accelerations of celestial bodies should be observed. Already Edmund Halley was interested in the existence of lunar accelerations (Halley, 1695). He writes, “And I could then pronounce in what Proportion the Moon’s Motion does Accelerate; which that it does, I think I can demonstrate, and shall (God willing) one day, make it appear to the Publick.”

Dunthorne (1739) derived the precession of the equinox $11^{\circ}32'$ in 810 years, i.e. $51''.2$ per year, and the obliquity $23^{\circ}35'$ from the separate observations of Ptolemy and al-Battani. After this, Dunthorne became interested in ancient astronomical observations such as Theon’s solar eclipses in AD 365. In 1749, he found after analyzing solar and lunar eclipses of the latter half of the fifteenth century that the calculated position of the Moon from the eclipses is $5'$ forward of the tabular values. He concluded that the moon’s mean motion is swifter than the tabular values. In other words, the motion of the Moon is accelerated.

The size of the solar acceleration is small compared to the lunar acceleration, and therefore its discovery was long after. Cowell (1905) finally found evidence from ancient eclipses for a significant solar acceleration.

2.2 Ginzel 1884, Fotheringham 1920, and Schoch 1926

In the late 19th century Hansen's Lunar Tables (1857) were used to calculate the Moon's positions, but it was known that the lunar acceleration obtained from ancient solar eclipses was different from the theoretical one. The coefficient of the T^2 term in the Moon's mean longitude with respect to the fixed stars obtained from ancient solar eclipses was fairly close to $10''$ (e.g. Celoria, 1877; Ginzel, 1884, 1918) but its theoretical value was about $6''$. Here T is the time counted in centuries. At that time it was not established yet what the cause of this discrepancy between the observations and the theory and it was being discussed if there existed errors in the lunar theory.

Fotheringham (1921) appreciated the publication of *Tables of the Motion of the Moon* by Brown (1919), which was resulted from Brown's stupendous work on the theory of the motion of the Moon. Brown (1915) confirmed the coefficient of the T^2 term in the Moon's mean longitude with respect to the fixed stars as $6''.03$. His theory was the most precise gravitational theory of the motion of the Moon, but even using this newest theory, the discrepancy in the lunar acceleration between the observations and the theory was not explained. Brown was confident that his gravitational theory was correct and therefore he felt no necessity of introducing artificial accelerating terms for the motion of the Moon. The diminution in the eccentricity of the Earth's orbit, which Laplace discovered, and the gravitational effect of the oblateness of the Earth's figure, which Stockwell discovered, were not enough to explain the observed lunar acceleration.

For the purpose of obtaining a precise acceleration of the Moon modern astronomical observations are of little value because the time span is too short to obtain the acceleration and moreover the Moon's motion is subject to unexplainable fluctuations, from which it is impossible to disentangle the effect of the acceleration.

Fotheringham (1920) analyzed the following 11 ancient solar eclipses and using also some ancient observations of lunar eclipses, occultations and equinoxes, he obtained the values of the coefficients of the T^2 terms in the mean longitudes with respect to the fixed stars as $+10''.8$ and $+1''.5$ for the Moon and for the Sun, respectively.

1	The eclipse of Babylon	−1062	7	31
2	The eponym canon eclipse	−762	6	15
3	The eclipse of Archilochus	−647	4	6
4	The eclipse of Thales	−584	5	28
5	The eclipse of Pindar	−462	4	30
6	The eclipse of Thucydides	−430	8	3
7	The eclipse of Agathocles	−309	8	15
8	The eclipse of Hipparchus	−128	11	20
9	The eclipse of Phlegon	+29	11	24
10	The eclipse of Plutarch	+71	3	20
11	The eclipse of Theon	+364	6	16

Schoch (1926) examined many ancient solar eclipses, as was cited by Fotheringham (1920), but he used only one observation of an appulse of Spica with the Moon to derive the coefficient of the T^2 term in the Moon's mean longitude with respect to the fixed stars as $11''.09$. The observation he used was the one made by Timocharis at Alexandria on -282 Nov. 9, when Spica in the morning touched the north cusp of the Moon after the Moon had risen over the horizon (Schoch, 1928). Schoch's value was important because his expression for the Moon's longitude was adopted by Neugebauer (1929) and was used in the tables by Tuckerman (1964), and the tables were used by some historians to examine the places where ancient eclipses were seen.

2.3 de Sitter 1927 and Spencer Jones 1939

De Sitter (1927) analyzed the observed positions of the Moon, the Sun, Mercury and Venus since the 17th century and showed that the observed fluctuations in their positions were proportional to their mean motions. The fluctuations in their observed positions could therefore be attributed to a variation of the adopted unit of time provided by the rotation of the Earth.

Spencer Jones (1939) reanalyzed the observed positions of the Moon, the Sun, Mercury, and Venus and established the evidence that the fluctuations were proportional to their mean motions and were therefore attributed to variation of the rotation of the Earth. He obtained expressions for the corrections to the mean longitudes of the Moon, the Sun, Mercury, and Venus, from which Clemence (1948) deduced the tidal acceleration term in the Moon's mean longitude to be $-11''.22T^2$, which was not obtained by gravitational theory. Clemence also obtained an expression for the corrections to the Moon's mean longitude based on Spencer Jones's results in order to obtain a strictly gravitational lunar ephemeris expressed in the same measure of time as defined by Newcomb's Tables of the Sun. Clemence called the time Newtonian time, which was later named Ephemeris Time. Since then obtaining the values of $\Delta T = ET - UT$ for various epochs and the value of the tidal acceleration coefficient of the Moon from observations has become one of the important purposes in positional astronomy. Note that ET means Ephemeris Time, which is a uniform measure of time, and UT means Universal Time, which is determined by the rotation of the Earth. Currently ET is replaced by TT, viz. Terrestrial Time, which is defined by $TAI + 32.184s$ where TAI is International Atomic Time. TT is considered to be continuous both in the rate and in the value with ET.

3 Modern determinations of ΔT

3.1 R.R. Newton 1970

Newton (1970) used ancient astronomical observations to determine the tidal acceleration in the Moon's orbital motion and the acceleration of the Earth's spin rate. The types of observation he used include equinoxes, solstices, places of totality or near-totality of solar eclipses, times of various phases of solar and lunar eclipses, magnitudes of partial solar and lunar eclipses, conjunctions, and occultations. Among them, records of solar eclipses with large magnitudes occupy a large fraction of those he analyzed and the numbers of

such solar eclipses are listed in Table 3.1. He assigned a reliability (weight) to each eclipse record in the form of numbers from 0 to 1 for his analyses.

Table 3.1. Large solar eclipses used by Newton

Places	No.	Periods in AD years
British Isles	7	664–1191
Babylonia + Assyria	2	–1062, –762
China	20	–708–1221
Europe	13	–50–1241
Mediterranean countries	27	–688–590

3.2 F.R. Stephenson, 1995, 1997

F.R. Stephenson collected many ancient observation data from various sources. In particular, he brought East Asian data into analyses. The numbers of data until 1600 he used are given in Table 3.2.

Table 3.2. The numbers of solar and lunar eclipses used

Places	Solar Ecl.	Lunar Ecl.
Babylonia + Assyria	14	74
China	70	25
Korea	2	0
Japan	2	0
Europe + Alexandria	47	8
Arab	25	29

There are a lot of observations in Asia other than China, but much of them were not incorporated. The analysis of the reliability and incorporation of these data are left to future authors.

The main result of Stephenson (1997) is represented in Fig. 1. Here, the abscissa denotes years and the ordinate stands for ΔT . In the figure, the solid curve is the expected variation of ΔT due to the angular momentum conservation of the Earth-Moon system. Various point marks represent timed lunar and solar eclipses from different periods and different countries. The dotted curve is the cubic spline ΔT curve. Interpretation due to Stephenson is that the scatters of the data points are very large, so that no short-term variations can be derived. The long-term variations of ΔT , then, should be smooth.

The scattered points are not around the solid curve but around the dotted curve. This provides evidence that there is a non-tidal component in the variation of the Earth's rotation. A possible our interpretation for it is that the sea level was higher and the inertial moment of the Earth was larger than those expected from neglecting the long-term variations of the global climates as we go back to the past. The observational tendency is explainable if ices and glaciers in the Arctic and the Antarctic regions were melted because of the warm climate toward a few thousand years ago.

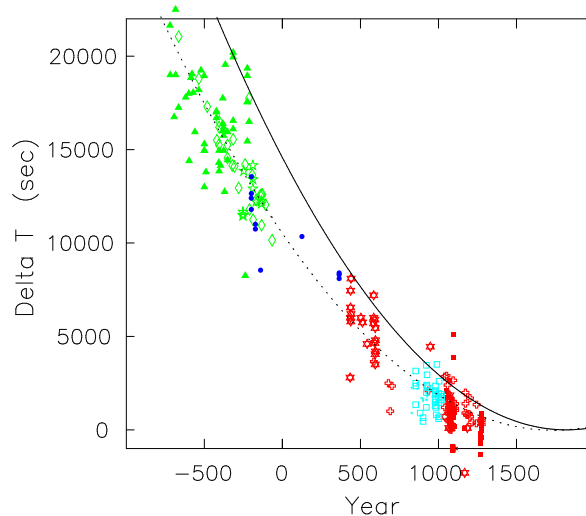


Figure 1: ΔT variations. The solid curve represents the ΔT curve predicted by conservation of angular momentum of the Earth-Moon system. The dotted curve is the cubic spline fitting to ΔT due to Stephenson (1997). Timed lunar and solar eclipses are highly weighted.

4 Tanikawa and Sôma since 2001

Tanikawa and Sôma started their career of study of the ΔT determinations in 2001 almost just after the publication of the celebrated textbook of Stephenson (1997). Sôma had been interested in this subject since his master's thesis in 1980. Our main purpose is to find short periodic variations of ΔT of the order of a few hundred years or less.

There were two motivations. The first one is that Stephenson's cubic spline curve may not reflect the reality because it is heavily smoothed. His result is strongly dependent on the timed lunar and solar eclipses. Partial eclipses are used because they are timed. Babylonian timed lunar eclipses give so dispersed points in ΔT . This seems to us that the accuracy of Babylonian clocks are very bad. The second motivation is that Asian total solar eclipses are well off the curve. This suggests that there are fluctuations of ΔT which the spline curve misses.

For brevity of description, we summarize the Guideline of our study. Most importantly, we determine the range of ΔT using contemporaneous plural records. According to the method of analysis of Stephenson (1997), it is clear that Stephenson regards each observational data as having large intrinsic uncertainty which sometimes amounts to a few thousand time seconds. Parameters to be determined from these records are ΔT and the acceleration $\dot{n}$ of the lunar motion where n is the lunar mean motion. These two parameters are considered to change slowly. The ΔT values may change secularly by at most 10 seconds each year even in two thousand years ago. The change of $\dot{n}$ is considered smaller. Then, it is reasonable that we can regard these parameters are constant within

ten years with uncertainty of, say, 100 seconds.

We, in principle, do not use lunar eclipse data, because we do not know the accuracy of ancient clocks. The accuracy of ancient clocks should be determined after we obtain good ΔT variations.

Now, we incorporate solar eclipse and lunar occultation data as many as possible. We try to use total and near total eclipses, eclipses at sunrise or sunset, lunar grazing occultations, and lunar occultations at moonrise or moonset. We use data observed at plural sites; we use contemporaneous observations; we analyze the data with the Sôma diagram which will be introduced in section 4.2.

4.1 Eclipses observed at plural sites

The number of solar eclipses observed at plural sites is small. However, these are more useful when observing sites are far apart. Deep eclipses are desirable. But this condition is not indispensable if the sites are far apart. We list them in Table 4.1. We show the maps of eclipse bands for the eclipses in BC188, AD873, and AD1415 in Fig. 2.

Table 4.1. Deep solar eclipses observed at plural sites.

Year	Month	Day	Magnitudes and places
BC	648.04.06		Total at Thasos or Paros; Sunset at Qufu
BC	188.07.17		Total at Rome; nearly total at Chang-an
AD	873.07.28		Annular at Nishapur; Annular at Kyoto
AD	968.12.22		Total at Constantinople; Total at Farfa
AD	1133.08.02		Total at many cities in Europe
AD	1183.11.17		Deep at Mizushima, Kaesong, and Lin-an
AD	1241.10.06		Total at Stade, Reichersburg, and Nile Delta
AD	1245.07.25		Annular at Kaesong; Deep at Kyoto
AD	1415.06.07		Total at Aizu (Japan), Neider Alteich, Prague, Wroclaw, Kobrin, and Mytho; Partial at Kyoto

4.2 Contemporaneous observations

There can be a lot of groups of contemporaneous observations. Here, we list representative examples in Table 4.2. Let us describe how to use these contemporaneous observations. In general, we fix the latitude and longitude of the observing site. For the fixed value of $\dot{n}$, we get the upper and lower bounds of ΔT for which the site is inside the totality or annularity band of the given eclipse. Then, we change the value of $\dot{n}$ by some amount and calculate the upper and lower bounds of ΔT for which the site is again inside the totality or annularity band. We repeat this process several times. After this process, we obtain nearly parallel curves in the $\dot{n}$ - ΔT -plane, which we call the Sôma diagram. The region sandwiched by these two curves is where the eclipse is total or annular at the observing site. As a starting value of $\dot{n}$ we take $-25''.826/\text{cy}$, which is the value intrinsic to the JPL

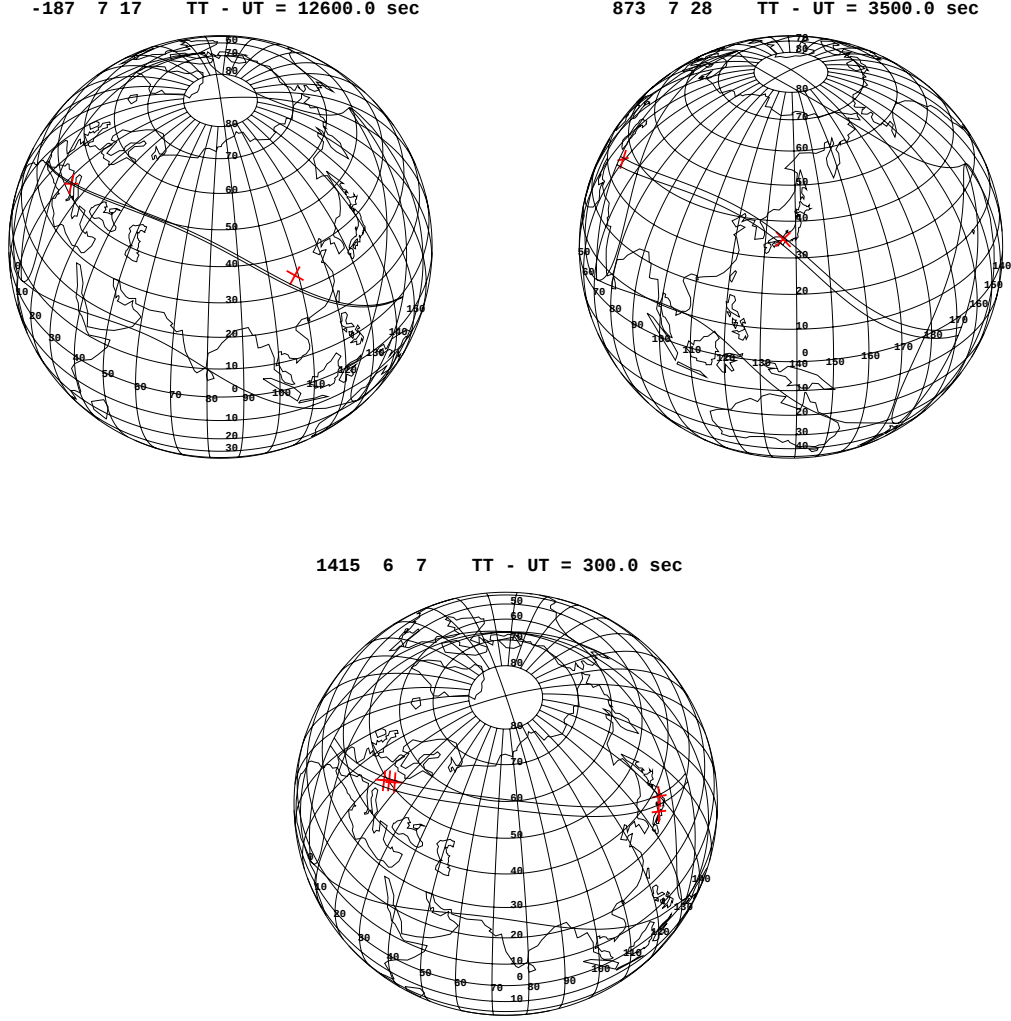


Figure 2: Eclipse bands of the three eclipses in BC 188, AD 873, and AD 1415. The eclipse of BC 188 was observed at Rome and Chang-an (長安). The eclipse of AD 873 was observed at Nishapur and Kyoto (京都). The eclipse of AD 1415 was observed at Aizu (会津), Neider Alteich, Prague, and Wroclaw as total. In Kyoto (京都), it was observed as partial.

lunar and planetary ephemerides DE405 and DE406 (Chapront et al. 2002). This value is consistent with the values of the more recent ephemerides DE430 ($-25''.82 \pm 0''.03/\text{cy}$) and DE431 ($-25''.80 \pm 0''.03/\text{cy}$), and agrees well with the value $-25''.858 \pm 0''.003/\text{cy}$ obtained from analyses of LLR observations (Chapront et al. 2002).

In the case of lunar grazing occultations of planets or bright stars, we generally determine the one boundary for which these objects are hidden by the lunar disk. So, we get either the upper or lower boundary of ΔT . For the sunset or sunrise eclipses, there can be two boundary curves. In fact, one boundary corresponds to the situation such that the sun is hidden by the moon, whereas the other boundary is such that eclipse itself is interrupted by the sunset or sunrise.

Table 4.2 Examples of contemporaneous observations.

BC	600.09.20	Total solar eclipse at Qufu
BC	584.05.28	Total solar eclipse at Qufu
BC	198.08.07	Annular solar eclipse at Chang-an
BC	188.07.17	Deep solar eclipse at Chang-an
BC	188.07.17	Total solar eclipse at Rome
BC	181.03.04	Total solar eclipse at Chang-an
AD	503.08.05	Occultation of Venus at Luoyang
AD	513.08.22	Occultation of Saturn at Luoyang
AD	516.04.18	Annular solar eclipse at Jiankang
AD	522.06.10	Total solar eclipse at Jiankang
AD	1124.08.11	Total solar eclipse at Novgorod
AD	1133.08.02	Total solar eclipse at seven cities in Europe
AD	1135.01.16	Partial solar eclipse at Lin-an
AD	1239.06.03	Total solar eclipse at eight cities in Europe
AD	1241.10.06	Total solar eclipse at two cities in Europe and a city in Egypt
AD	1245.07.25	Annular solar eclipse at Kaesong Deep at Kyoto

We first show the usage of the Sôma diagram with four deep solar eclipses at around BC 188 observed at Chang-an and Rome (see Table 4.2). As shown in Fig 3, there are four pairs of curves corresponding to three observations at Chang-an and one observation at Rome. The meaning of the abscissa is that 0.0 corresponds to $\dot{n}_0 = -25''.826/\text{cy}^2$ and -2.0 corresponds to $\dot{n} = \dot{n}_0 - 2''.0/\text{cy}^2$. Thus, the abscissa values are corrections to the current value of $\dot{n}$.

The second example of the usage of the Sôma diagram is shown with lunar occultations (see the third group of the observations in Table 4.2). The first two data are the lunar occultation of Venus on Aug. 5, AD 503 at Luoyang, and the lunar occultation of Saturn on Aug. 22, AD 513 also at Luoyang. The remaining two events are the solar eclipse on

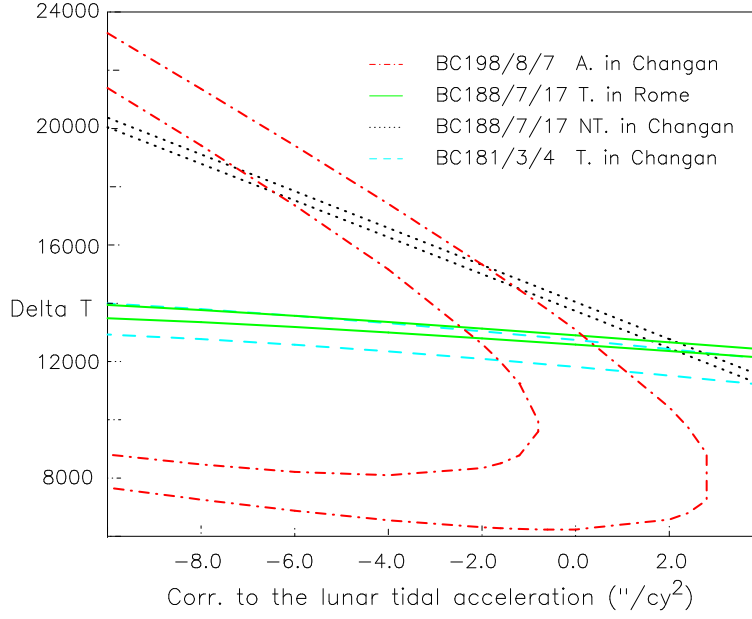


Figure 3: **The Sôma diagram for the years BC 198, 188, and 181.**

Apr. 18, 516 at Jiankang, and the solar eclipse on June 10, 522 also at Jiankang. The maps of the events are shown in Fig. 4. The two maps at the top show that the lunar occultations took place in the early morning and in the late evening, respectively. These situations give us the upper and lower boundaries of ΔT for which the lunar occultations were seen above the horizon. The bottom two figures are for the total and annular solar eclipses, respectively, and therefore the meaning of the maps is clear.

The Sôma diagram for these events is shown in Fig. 5. The two curves with upward arrows and downward arrows are for the two lunar occultations. This figure shows that the result from two lunar occultations is perfectly consistent with that from two solar eclipses. This fact shows that occultation data are useful in some cases for our research.

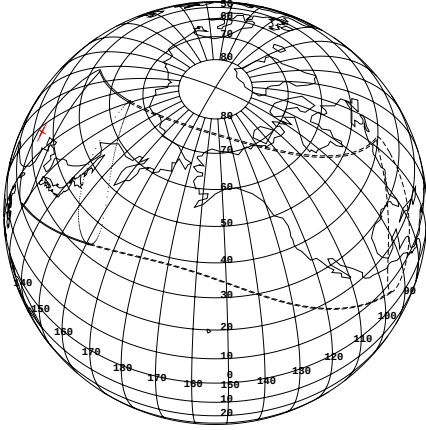
4.3 Some results

We have determined the values of ΔT for some selected periods (Tanikawa & Sôma 2004, 2015; Tanikawa et al. 2010). We plot the results in Fig. 6. We enlarge of the boxed area of the above figure in Fig. 7.

Let us talk about the warm and cold climates deduced from the long-term variations of ΔT as shown in Fig. 7. In the warm periods, ice-caps in the Arctic and Antarctic regions melt, water comes to the equatorial regions, the inertial moment of the Earth increases, and the angular velocity of the Earth rotation decreases. In the periods of the slow rotation, the ΔT curve becomes (relatively) flat. In the opposite case, that is, the ΔT curve is steep, the climate is cold.

We will talk about the meaning of Fig. 7 more precisely in a separate paper.

Occultation of Venus on 503 8 5



Occultation of Saturn on 513 8 22

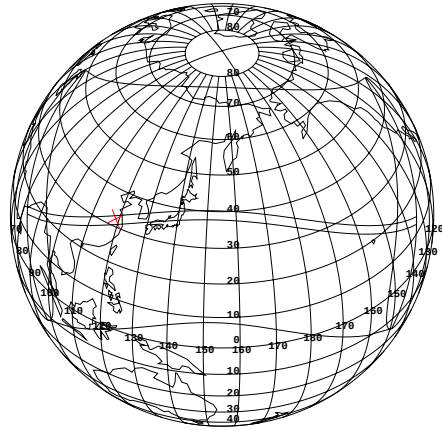
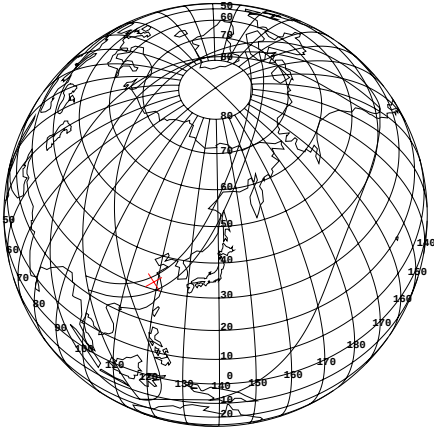
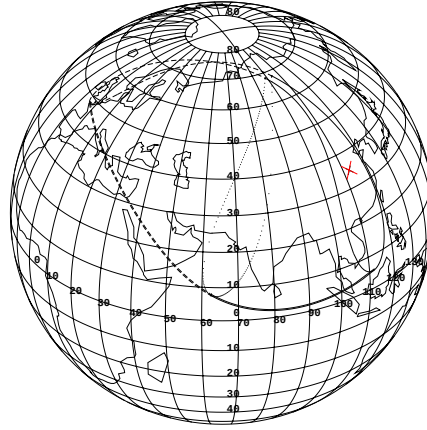


Figure 4: The occultation of Venus on AD 503.08.05 at Luoyang (洛陽). The occultation of Saturn on AD 513.08.22 at Luoyang (洛陽). $\Delta T = 4500$ sec. Moonrise. Eclipses on 516.04.18 at Jiankang (建康). Eclipses on 522.06.10 at Jiankang (建康). $\Delta T = 4500$ sec.

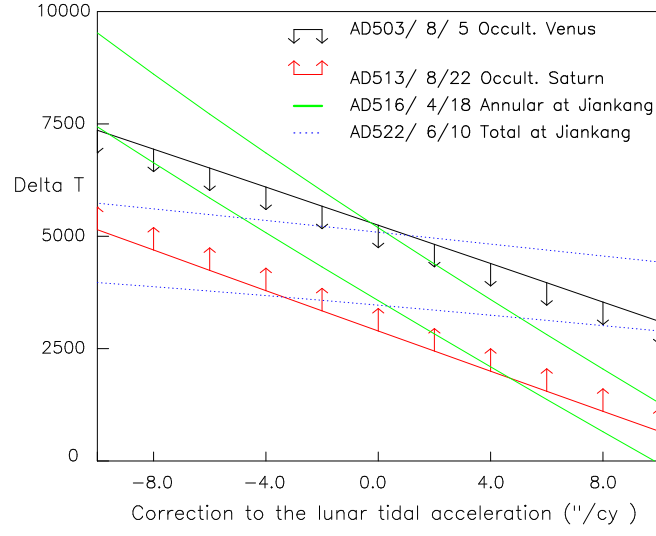


Figure 5: The range of ΔT from the Sôma Diagram for the years AD 503, 513, 516, and 522.

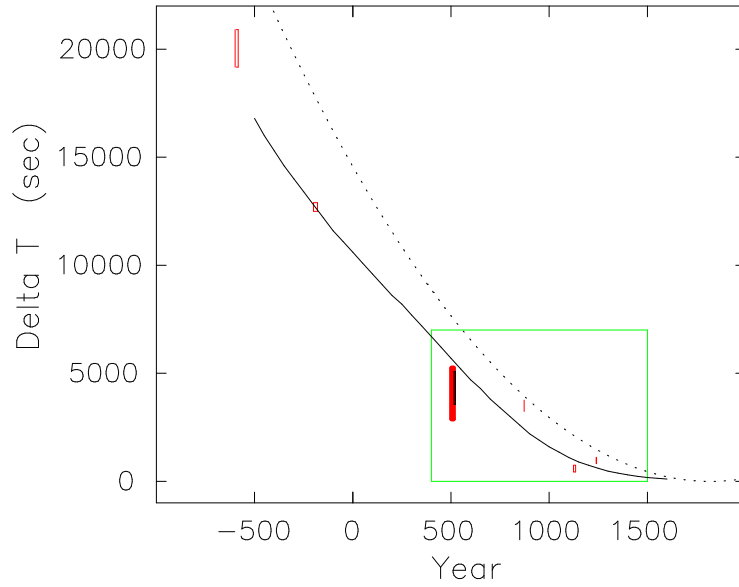


Figure 6: ΔT variations. The dotted curve corresponds to the conservation of angular momentum of the Earth–Moon system. Solid curve: the cubic spline fitting due to Stephenson (1997). Vertical bars show our results.

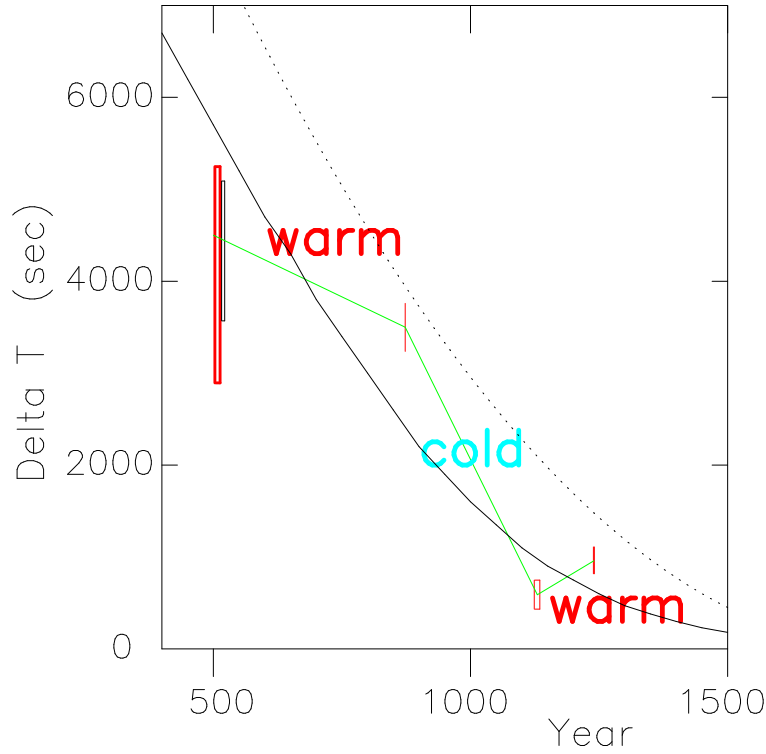


Figure 7: ΔT variations and Climate.

5 Concluding remarks

Figs. 6 and 7 suggest that short-term variations, shorter than a few hundred years, of ΔT exist. In order to draw the figures we have used reliable contemporaneous records. The figures show that shorter variations in the ΔT curve do exist.

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